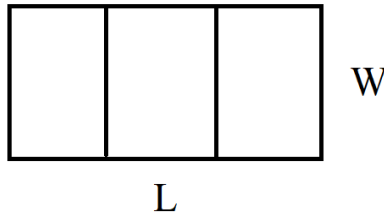


Exercise 87

Find the dimensions of the rectangular corral producing the greatest enclosed area split into 3 pens of the same size given 500 feet of fencing.

Solution

Draw a schematic of the rectangular corral, labelling the length and width as L and W , respectively.



The perimeter is the sum of the lengths.

$$\begin{aligned}P &= L + L + W + W + W + W \\ &= 2L + 4W\end{aligned}$$

It's given to be 500 feet.

$$500 = 2L + 4W$$

Solve for L .

$$500 - 4W = 2L$$

$$\frac{1}{2}(500 - 4W) = L$$

$$L = 250 - 2W$$

Write the formula for the area, substitute the result for the length, and complete the square to write the quadratic function in vertex form.

$$\begin{aligned}A &= LW = (250 - 2W)W \\ &= 250W - 2W^2 \\ &= -2(W^2 - 125W) \\ &= -2[(W^2 - 125W + 62.5^2) - 62.5^2] \\ &= -2[(W - 62.5)^2 - 3906.25] \\ &= -2(W - 62.5)^2 + 7812.5\end{aligned}$$

Therefore, the maximum area is $A = 7812.5 \text{ ft}^2$, which occurs when $W = 62.5 \text{ ft}$ and $L = 250 - 2(62.5) = 125 \text{ ft}$.